From E_{11} to Supergravity

Teake Nutma
Centre for Theoretical Physics, University of Groningen

Introduction

Supersymmetry, a symmetry between bosons and fermions, is one of the viable extensions of the Standard Model. When demanding that this symmetry is coordinate-dependent one automatically obtains a *supergravity* (SUGRA).

The mother of all supergravities is the 11-dimensional SUGRA. For some time now it has been conjectured [1] that the infinite-dimensional Lie algebra of the group E_{11} is its underlying symmetry.

Although D=11 SUGRA does not describe the world we live in, it is nonetheless interesting because it yields all maximal (ungauged) SUGRAS in lower dimensions upon compactification over a torus.

Gaugings can be introduced by either gauging directly in lower dimensions [2], or by flux compactifications of D=11 SUGRA. However, the latter is not able to reproduce all possible direct gaugings.

It has been shown that a particular decomposition of the low-level generators of E_{11} correspond to the physical fields of 11-dimensional supergravity [3], and other decompositions to the IIA and IIB SUGRAS in 10 dimensions [4].

This raises interesting questions ...

Research questions

- Can the field content of all $3 \leq D \leq 11$ maximal SUGRAS be obtained using various decompositions of E_{11}?
- And if so, can some higher-level generators give rise to fields that do not propagate, but nonetheless deform (i.e. gauge) the theory?

From Dynkin diagrams to Lie algebras

A compact way to represent a Lie algebra is the *Dynkin diagram*. It encodes the Cartan matrix A:

$$A_{ij} = \begin{cases} 2 & \text{if } i = j, \\ -1 & \text{if nodes } i \text{ and } j \text{ are connected,} \\ 0 & \text{otherwise.} \end{cases}$$

Assign to every i^{th} node three $SL(2)$ generators:

h_i : Abelian (Cartan) subalgebra
e_i : Positive step operator
f_i : Negative step operator

They satisfy

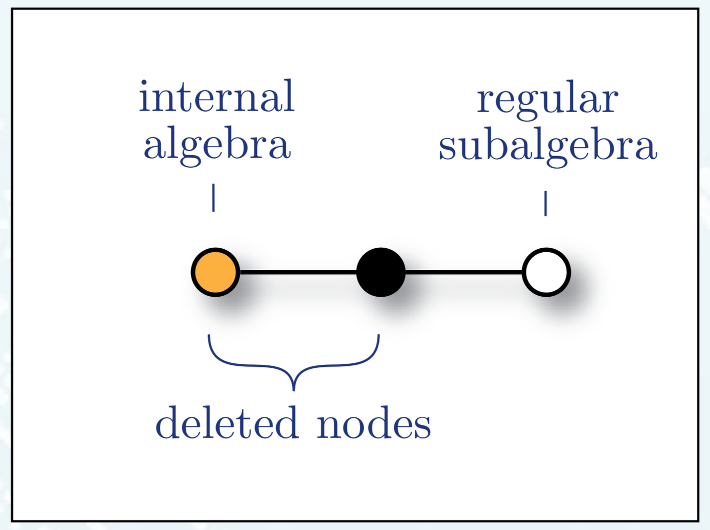
$$\begin{aligned} [h_i, h_j] &= 0, \\ [h_i, e_j] &= A_{ij} e_j, \\ [h_i, f_j] &= -A_{ij} f_j, \\ [e_i, f_j] &= \delta_{ij} h_i. \end{aligned}$$

The full Lie algebra is then generated by all multiple commutators of e's and fs, restricted by

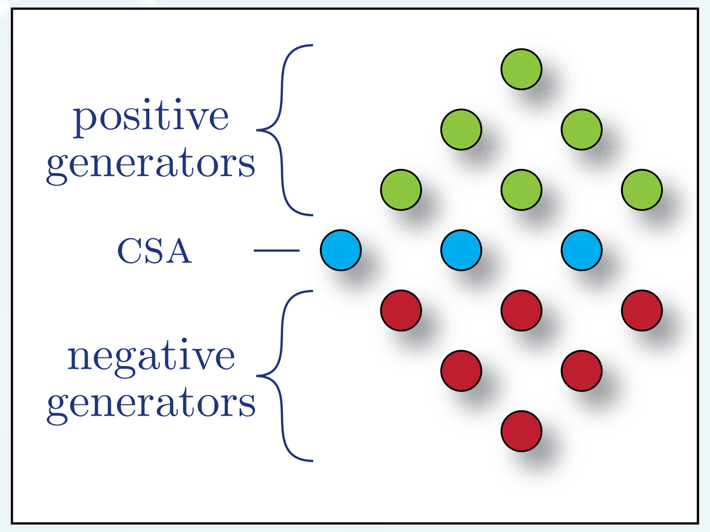
$$\begin{aligned} (\mathrm{ad}_{e_i})^{1-A_{ij}} e_j &= 0, \\ (\mathrm{ad}_{f_i})^{1-A_{ij}} f_j &= 0. \end{aligned}$$

Easy example: $SL(4)$

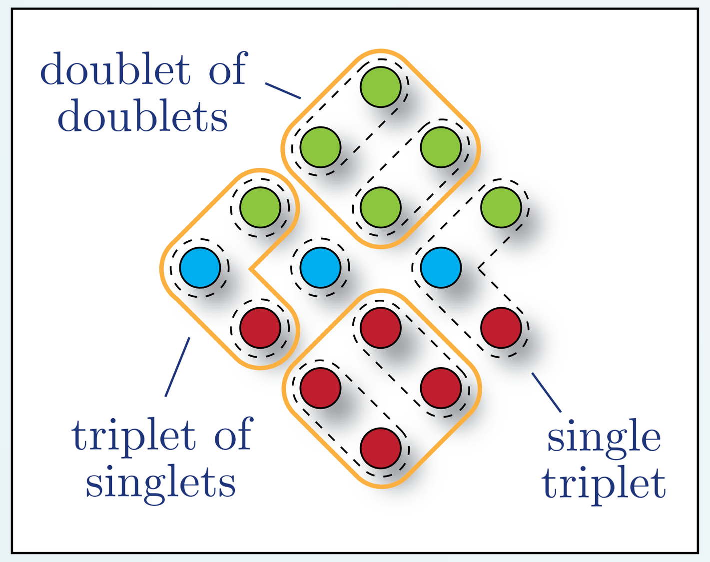
Two nodes in the diagram of $SL(4)$ are 'deleted'. The node with no connections to the regular subalgebra forms the internal algebra.



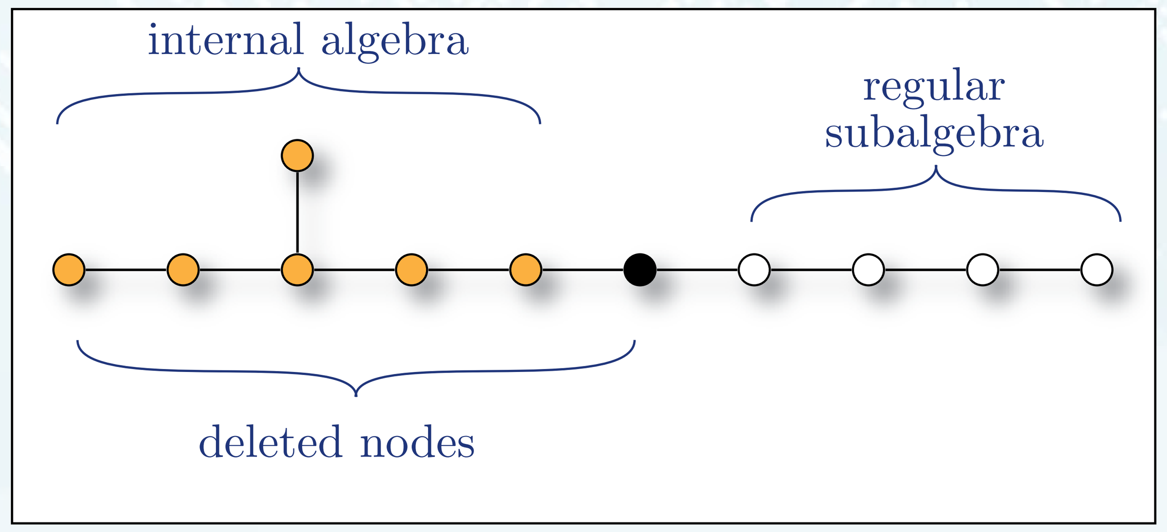
The Dynkin diagram gives rise to 15 generators: 6 positive, 6 negative, and 3 belonging to the Cartan subalgebra (CSA).

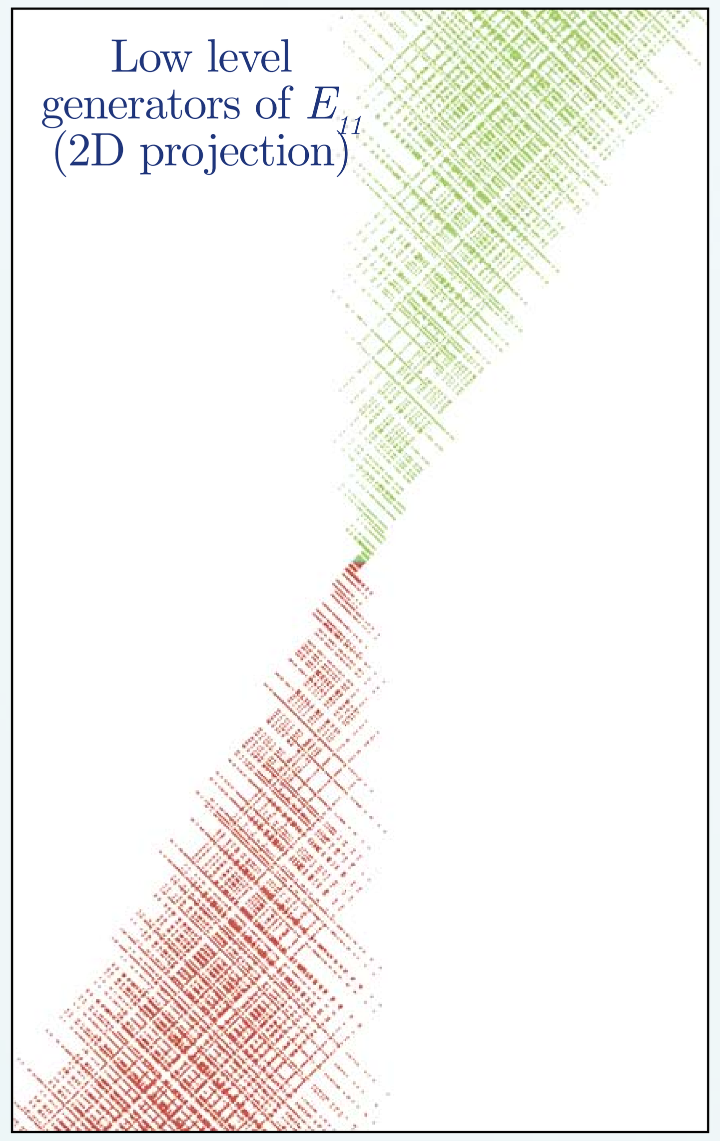


The 15 generators can be divided into representations of the regular (dotted lines) and internal (orange lines) subalgebra.



The real deal: E_{11}





The Dynkin diagram of E_{11} can be decomposed in the same way. This gives rise to a similar ordering of the generators.

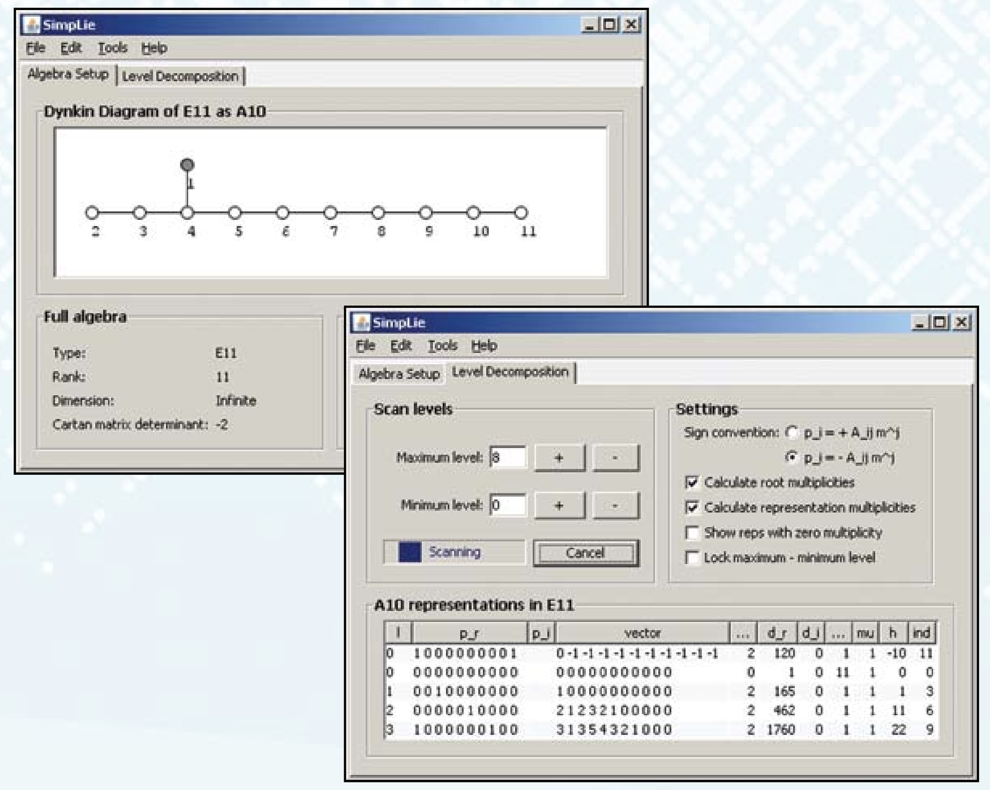
Problem:
How do you order an infinite amount of generators?

Solution:
Let the computer figure it out!

SimpLie

To speed up the decomposition of the E_{11} generators we have written a Java-based computer program, *SimpLie*.
From the user interface, one can:

- Create any simply-laced Dynkin diagram and 'delete' arbitrary nodes.
- Automatically scan for subalgebra representations at specified levels.



http://strings.fmns.rug.nl/SimpLie

Results

Using SimpLie, we have found [5]:

- The field content of all $3 \leq D \leq 11$ maximal SUGRAS as generators of E_{11}.
- Non-propagating 'de-form' and 'top-form' fields, which seem to have a close link to methods used to gauge these supergravities.

The de-forms and top-forms are not required by the supersymmetry algebra, and thus it cannot predict them. Our approach, and that of [6], shows that E_{11} can.

E-mail: t.a.nutma@rug.nl

[1] West, arXiv:hep-th/0104081
[2] De Wit, Samtleben, Trigiante, arXiv:hep-th/0311225
[3] Damour, Henneaux, Nicolai, arXiv:hep-th/0207267
[4] Kleinschmidt, Schnakenburg, West, arXiv:hep-th/0309198
[5] Bergshoeff, De Baetselier, Nutma, arXiv:0705.1304
[6] Riccioni, West, arXiv:0705.0752